

CLASSIFICATION OF INTERFACIAL FORCES, ACTING ON SOLID PARTICLES IN TECHNOLOGIES OF ADVANCED METALLIC MATERIALS

George Kaptay

University of Miskolc, Department of Physical Chemistry, Miskolc, Hungary

Abstract – The production of advanced metallic materials is influenced by different interfacial forces, acting on solid particles, situated in liquid metals. In the present paper a classification of all possible interfacial forces is given. The interfacial forces of different origin are derived in a generic way. Equations are presented for the simplified case of spherical particles.

Keywords: interfacial forces, solid particles, liquid metals

1. INTRODUCTION

Solid particles present in all technologies of producing advanced metallic materials. In primary metallurgy and metal casting the particles are usually unwanted inclusions, to be effectively removed from the liquid metal to produce 'clean metals'. In metal matrix composites (MMCs) particles play a vital role of the reinforcement phase, while in liquid metallic foams particles are responsible for foam stabilization. In all these technologies particles can occur in different situations under the influence of different types of interfacial forces.

The goal of the present paper is to discuss all different possible types of interfacial forces in a generic way. Interfacial forces will be classified into six categories. Therefore, if materials engineers want to understand each other, the simple term 'interfacial force' is not sufficient. In the present paper specific names will be suggested to be used for different types of interfacial forces. Equations will be derived for a simplest case of spherical solid particles.

2. THE GENERAL EQUATION FOR THE INTERFACIAL FORCES

Imagine a system, consisting of any number of phases. Let us consider the interfacial force, acting at an arbitrary chosen solid phase in an arbitrary chosen direction, x . The equation for the interfacial force can be obtained in the following way. First, the total interfacial energy of the system should be described as function of movement of the chosen solid phase in direction x , with an infinitely low velocity:

$$G_{\sigma} = \sum_{A,B} A_{A/B} \cdot \sigma_{A/B} \quad (1)$$

where A and B are different phases, $\sigma_{A/B}$ is the interfacial energy between phases A and B (J/m^2), $A_{A/B}$ is the interfacial area between the phases A and B (m^2) and G_{σ} is the total interfacial energy of the system (J).

Then, following Newton and Gibbs, the interfacial force, acting on the chosen phase in direction x can be obtained as a derivative of G_{σ} by x , taken with a negative sign:

$$F_{\sigma,x} = -\frac{dG_{\sigma}}{dx} \quad (2)$$

The negative sign follows from the sign convention of thermodynamics, saying that spontaneous processes take place along paths with some negative change of the Gibbs energy. When the interfacial force appears to be positive from Eq.(2), its direction (as a vector) is the same as that of x , and vice versa.

Substituting Eq.(1) into Eq.(2) and performing the derivation, the following general equation can be obtained:

$$F_{\sigma,x} = -\sum_{A,B} \sigma_{A/B} \cdot \frac{dA_{A/B}}{dx} - \sum_{A,B} A_{A/B} \cdot \frac{d\sigma_{A/B}}{dx} \quad (3)$$

As follows from Eq.(3), the interfacial force appears for two reasons: due to the change of any interfacial energy, or of any interfacial area in the system, while the studied solid phase moves along the path x . If none of these quantities change along the path of the solid phase, there is no interfacial force, acting on this phase.

Eq.(3) is applicable only if along the considered path x the interfacial Gibbs energy of the system is a continuous function of x , i.e. if there is no break on G_{σ} as function of x .

An alternative way to derive the interfacial force is to use the Young-Laplace method. However, this method is complicated to use correctly, what leads to frequently appearing incorrect results, published in the literature.

3. THE CURVATURE INDUCED INTERFACIAL FORCE (THE LAPLACE EQUATION)

Let us consider a small, spherical phase A of radius x in an infinitely large, homogeneous fluid phase B . Let the radius of the sphere increase infinitely slowly, without changing the bulk energies of the system. Then, the only A/B interface of the system will have a constant interfacial

energy, but a variable interfacial area. Therefore, Eq.(3) will give: $F_{\sigma,x} = -\sigma_{A/B} \cdot (dA_{A/B}/dx)$. Substituting $A_{A/B} = 4\pi x^2$ for a sphere, performing the derivation and dividing the interfacial force by the same expression $A_{A/B} = 4\pi x^2$, the following equation for the interfacial pressure is obtained:

$$P_{\sigma,x} \equiv \frac{F_{\sigma,x}}{A_{A/B}} = -\frac{2 \cdot \sigma_{A/B}}{x} \quad (4)$$

Eq.(4) is the classical Laplace equation for a sphere. The minus sign indicates that the interfacial pressure acts from outside towards the centre of the sphere, along the interface of the small spherical phase. This ‘‘curvature induced interfacial force’’ acts only because of the existence of the curvature. Eq.(4) is derived here for two reasons. First, to demonstrate that the well-known Laplace equation follows from the general Eq.(3), so Eq.(4) partly validates Eq.(3). Second, to show that the Laplace equation is not the basis for all other interfacial forces, it is just one of the consequences of a most general Eq.(3).

4. THE INTERFACIAL GRADIENT FORCE

Let us consider a small phase A of a constant size and shape, moving inside a large, inhomogeneous phase B. The temperature- and composition-inhomogeneities of phase B cause the variation of the interfacial energy along path x:

$$\frac{d\sigma_{A/B}}{dx} = \frac{d\sigma_{A/B}}{dT} \cdot \frac{dT}{dx} + \sum_i \frac{d\sigma_{A/B}}{dx_i} \cdot \frac{dx_i}{dx} \quad (5)$$

where x_i is the mole fraction of component i in phase B. As along the path of phase A its interfacial area $A_{A/B}$ remains constant, only the second term of Eq.(3) will be effective in this case. Then, the ‘‘interfacial gradient force’’ will equal:

$$F_{\sigma,x} = -A_{A/B} \cdot \frac{d\sigma_{A/B}}{dx} \quad (6)$$

The ‘minus’ sign in Eq.(6) indicates that the force is pointed towards the region of phase B with lower interfacial energy (usually of higher temperature and with higher concentration of interface active components).

If the temperature gradient or the composition gradient is not active along the whole interface of phase A, only that part of the interfacial area $A_{A/B}$ should be taken into account in Eq.(6), along which the gradients are active. For example, behind a moving spherical particle A the eddies might mix phase B, and thus the composition gradient is active only along the first half of the sphere. When a particle of low heat conductivity is placed in a temperature gradient, the temperature gradient might be shielded behind the particle, so again only half of the particle interface will be active. Then, for spherical particle of radius r Eq.(6) becomes:

$$F_{\sigma,x} = -2 \cdot \pi \cdot r^2 \cdot \frac{d\sigma_{A/B}}{dx} \quad (6.a)$$

The interfacial gradient force was introduced into the metallurgical literature by Mukai and Lin [1], who also performed very precise experimental measurements [2]. Eqs.(6-6.a) were derived in [3, 4], although the same equation was used in an implicit form much earlier [5, 6] (see also experimental verification [7-9]).

5. THE INTERFACIAL CAPILLARY FORCE

Let us consider a small phase 3, situated at the interface of two fluid phases 1 and 2, and transferring with an infinitely small velocity from phase 2 into phase 1. Let us consider the situation when phase 3 is situated at the depth of immersion x into phase 1. During this process the three interfaces will gradually change, hence Eq.(1) will include three terms:

$$G_\sigma = A_{1/2} \cdot \sigma_{1/2} + A_{1/3} \cdot \sigma_{1/3} + A_{2/3} \cdot \sigma_{2/3} = (A_{1/2}^o - \Delta A_{1/2}(x)) \cdot \sigma_{1/2} + A_3 \cdot \sigma_{2/3} + A_{1/3}(x) \cdot (\sigma_{1/3} - \sigma_{2/3}) \quad (7)$$

where $A_{1/2}^o$ is the initial fluid/fluid interface area, without phase 3, $\Delta A_{1/2}(x)$ is the x -dependent fluid/fluid interface area, covered by phase 3, A_3 is the total surface area of phase 3 and $A_{1/3}(x)$ is the x -dependent interfacial area between phases 1 and 3.

Substituting Eq.(7) into Eq.(2) and performing the derivation, the general equation can be derived for the ‘‘interfacial capillary force’’, acting on phase 3, in a perpendicular direction to the fluid/fluid interface:

$$F_{\sigma,x} = \sigma_{1/2} \cdot \frac{d\Delta A_{1/2}(x)}{dx} + (\sigma_{2/3} - \sigma_{1/3}) \cdot \frac{dA_{1/3}(x)}{dx} \quad (8)$$

Eq.(8) is generally valid for any shape of phase 3, neglecting the effect of curvature of the fluid/fluid interface along the surface of phase 3. When phase 3 is a solid cylindrical capillary with an inner radius of r and constant wall thickness, situated perpendicular to the fluid/fluid interface, the geometrical relations are as follows: $\Delta A_{1/2} = \text{const}$, $A_{1/3} = 2r\pi x$. Substituting it into Eq.(8):

$$F_{\sigma,x} = 2 \cdot r \cdot \pi \cdot (\sigma_{2/3} - \sigma_{1/3}) \quad (9.a)$$

In a particular case, when the fluid/fluid interface is a liquid/gas interface, Eq.(9.a) can be written as follows, if the Young equation is taken into account:

$$F_{\sigma,x} = 2 \cdot r \cdot \pi \cdot \sigma_{l/g} \cdot \cos \Theta \quad (9.b)$$

where $\sigma_{l/g}$ is the surface tension of the liquid metal, Θ is the contact angle of the liquid metal on a solid ceramic (wall) in a gas atmosphere. Combining Eq.(9.b) with a gravity force, the classical equation of the capillary rise can be obtained. That is why the interfacial forces in this sub-chapter are called in a generic way as ‘‘interfacial capillary forces’’.

When phase 3 has a spherical shape, the following geometrical relations are valid: $\Delta A_{1/2} = 2 \cdot r \cdot \pi \cdot x - \pi \cdot x^2$, $A_{1/3} = 2 \cdot r \cdot \pi \cdot x$. Substituting these equations into Eq.(8) and performing the derivation, the following general equation is obtained:

$$F_{\sigma,x} = 2 \cdot r \cdot \pi \cdot \sigma_{1/2} \cdot \left[1 + \frac{\sigma_{2/3} - \sigma_{1/3}}{\sigma_{1/2}} - \frac{x}{r} \right] \quad (10.a)$$

For the particular case when a solid particle 3 is situated on the liquid/gas interface, Eq.(10.a) becomes:

$$F_{\sigma,x} = 2 \cdot r \cdot \pi \cdot \sigma_{1/g} \cdot \left(1 + \cos \Theta - \frac{x}{r} \right) \quad (10.b)$$

Eq.(10.b) was derived by different authors in an independent way several times [10-12]. It should be mentioned that during the same period there is a larger amount of papers with different incorrect equations.

In the absence of other forces, the particle reaches its equilibrium at the liquid/gas interface when $F_{\sigma,x} = 0$. Substituting this condition into Eq.(10.b), the equilibrium depth of immersion of the particle equals:

$$x_{eq} = r \cdot (1 + \cos \Theta) \quad (11)$$

As follows from Eq.(11), the contact angle defines solely the equilibrium relative position of a small particle at the liquid/gas interface, if $0^\circ < \Theta < 180^\circ$. Substituting Eq.(11) into Eq.(10.b), one can write:

$$F_{\sigma,x} = 2 \cdot \pi \cdot \sigma_{1/g} \cdot (x_{eq} - x) \quad (10.c)$$

As follows from Eq.(10.c), the interfacial (capillary) force always tends to return the particle into its equilibrium position, if its actual position is different from that. Moreover, the interfacial force increases with the deviation of the actual position of the particle from its equilibrium position. In other words, the interfacial capillary force acts as an “elastic stabilizer”, or as a “spring”. This unique property of this force is what is behind the ability of small solid particles to stabilize liquid (metallic) foams [13-17].

When a spherical solid particle of density ρ_s is situated on the liquid/gas interface of a liquid with lower density ($\rho_l < \rho_s$), the particle will be detached from the interface above a certain critical radius, r_{cr} . From the balance of the interfacial capillary force (10.b) at $x = 2r$ with the gravity and buoyancy forces this critical size can be found [12]:

$$r_{cr} = 1.22 \cdot \sqrt{\frac{\sigma_{1/g} \cdot (1 - \cos \Theta)}{(\rho_s - \rho_l) \cdot g}} \quad (12)$$

where $g = 9.81 \text{ m/s}^2$ (on Earth), the gravity constant. Eq.(12) describes the critical radius better than the earlier equations [18-19], as shown by comparison with experiments [20-21].

The equations derived above are based on the assumption that the horizontal liquid/gas interface is not disturbed by the solid particle. In reality, however, it is usually not the case. Although the liquid meniscus along the particle interface does not alter significantly the value of the interfacial capillary force, it will lead to the so-called interfacial meniscus force (see below). That is why the condition of a horizontal meniscus around a spherical particle should be found. It will take place, if the equilibrium position of the particle due to the balance of the gravity and buoyancy forces from one hand, and due to the interfacial capillary force from the other hand, will equal. Combining Eq.(11) with the well known equations for the gravity and buoyancy forces, this equality will be ensured, if the dimensionless density of the particle ρ^* will equal a certain value ρ^*_{flat} , defined solely by the contact angle:

$$\rho^*_{flat} = \frac{1}{4} \cdot (1 + \cos \Theta)^2 \cdot (2 - \cos \Theta) \quad (13)$$

where $\rho^* \equiv (\rho_s - \rho_g) / (\rho_l - \rho_g) \cong \rho_s / \rho_l$ with ρ_g – the density of the gas.

6. THE INTERFACIAL ADHESION FORCE

Let us consider two, solid particles 2 and 3, within a smallest distance x from each other, within a homogeneous fluid phase 1. The two particles will attract or repulse each other from a distance due to the so called “interfacial adhesion force”. This force is mainly responsible for agglomeration (clustering) of solid particles in liquid metals.

In this case the total interfacial energy G_σ contains two terms, corresponding to the fluid/solid 1/2 and 1/3 interfaces. If the interfacial energies are treated as constants, and the particles are considered rigid, G_σ will not be a function of x , and thus the interfacial adhesion force will be zero (see Eq.2). However, when the two particles appear close to each other, they will influence the energy of the surface atoms of each other, i.e. $\sigma_{1/2}$ and $\sigma_{1/3}$ become functions of x , and thus the interfacial adhesion force will appear. In this case, the effective surface areas of the two particles, influenced by each other will always equal, i.e. $A = A_{1/2}^* = A_{1/3}^*$. Then, from the second term of Eq.(3):

$$F_{\sigma,x} = -A \cdot \left(\frac{d\sigma_{1/2}(x)}{dx} + \frac{d\sigma_{1/3}(x)}{dx} \right) \quad (14)$$

The distance-dependence of the interfacial energies can be described as [22-23]:

$$\sigma_{1/2}(x) = \sigma_{1/2} + (\sigma_{2/3} - \sigma_{1/2}) \cdot \left(\frac{d}{d+x} \right)^2 \quad (15.a)$$

$$\sigma_{1/3}(x) = \sigma_{1/3} + (\sigma_{2/3} - \sigma_{1/3}) \cdot \left(\frac{d}{d+x} \right)^2 \quad (15.b)$$

where d – is the diameter of the atom in the liquid metal.

Now, for simplicity, let us apply Eq.(14) for two, flat particles with parallel areas of A , facing each other. Then, after substituting Eq-s (15.a-b) into Eq.(14) and performing the derivation, the result is:

$$F_{\sigma,x} = 2 \cdot A \cdot \Delta\sigma \cdot \frac{d^2}{(d+x)^3} \quad (16.a)$$

with

$$\Delta\sigma = 2 \cdot \sigma_{2/3} - \sigma_{1/2} - \sigma_{1/3} \quad (17)$$

Eq.(16.a) has been well-known in the literature [24-26]. When two spherical particles with radii r_2 and r_3 interact from a distance, Eq.(16.a) transforms as [25-28]:

$$F_{\sigma,x} = 2 \cdot \pi \cdot \frac{r_2 \cdot r_3}{r_2 + r_3} \cdot \Delta\sigma \cdot \frac{d^2}{(d+x)^2} \quad (16.b)$$

Although the classical equations (16.a-b), derived in [24, 27, 28] remain valid, the equation for parameter $\Delta\sigma$, derived in [22-23] as shown in Eq.(17) is significantly different from the classical result of Hamaker [28]. The validity of Eq.(17) is confirmed by its comparison with the results of pushing/engulfment experiments in liquid metallic systems (see [23] and references thereof).

As follows from Eq-s (16-17), two, chemically identical particles (phase 2 = phase 3, i.e. $\sigma_{2/3} = 0$, $\sigma_{1/2} = \sigma_{1/3}$), always attract each other ($\Delta\sigma < 0$), while two, chemically different particles can attract, or repulse each other, depending on the sign of parameter $\Delta\sigma$. The coagulation of two solid particles in liquid metals has been discussed in [29-30], based on Eq-s (16.b, 17).

7. THE INTERFACIAL MENISCUS FORCE

Let us consider two spherical particles 3 and 4 at the interface of two fluid phases 1 and 2 (see Fig. 1). When $\rho^* = \rho^*_{flat}$ (see Eq.(13)) for two, identical particles (see Fig.1.b), the meniscus around both the particles is flat, and therefore the interfacial energy G_σ of the system will not change as function of their distance. Thus, as follows from Eq.(2), there is no interfacial (meniscus) force between them (however, at very small distance the interfacial adhesion force will be active – see Eq-s (16-17)).

For the majority of particle/liquid combinations the condition $\rho^* = \rho^*_{flat}$ is not fulfilled, and therefore the meniscus around the particles is not flat (see Fig-s 1.a and 1.b). Therefore, the shape of the meniscus between two neighbouring particles will be a function of their separation, i.e. the total interfacial energy G_σ of the system will be also the function of x . As a consequence, in all cases when $\rho^* \neq \rho^*_{flat}$, the “interfacial meniscus force” will arise, being responsible for the coagulation of particles on liquid metal surfaces. From Eq.(3) the following equation can be written:

$$F_{\sigma,x} = \sigma_{1/2} \cdot \left(\frac{dA_{2/3}}{dx} \cdot \cos\Theta_{3/2/1} + \frac{dA_{2/4}}{dx} \cdot \cos\Theta_{4/2/1} - \frac{dA_{12}}{dx} \right) \quad (18)$$

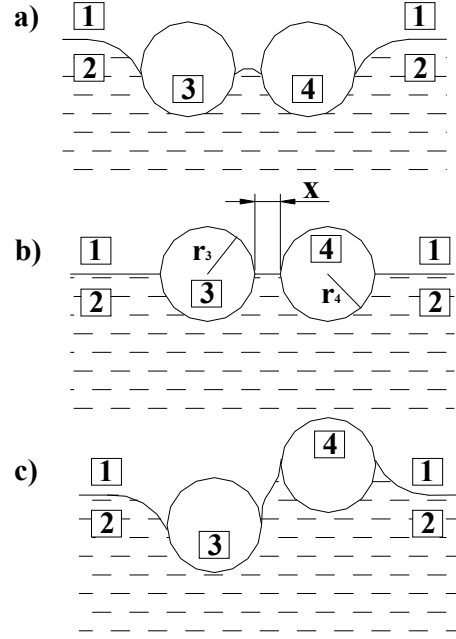


Fig.1. Two, spherical particles 3 and 4 at the interface of two fluid phases 1 and 2 with identical particles (a), with identical neutral particles (b) and with different particles (c)

In order to apply Eq.(18), first the interfacial areas $A_{2/3}$, $A_{2/4}$ and $A_{1/2}$ should be derived as function of x . This difficult task can be solved only numerically [31-33]. The analytical solution exist only when certain approximations are used. The approximated equation of [34] for the interfacial meniscus force between particle 3 with mass m_3 and particle 4 with mass m_4 can be re-written as:

$$F_{\sigma,x} \cong - \frac{81 \cdot g^2}{32 \cdot \pi} \cdot \frac{m_3 \cdot m_4}{x \cdot \sigma_{1/2}} \cdot (\rho_3^* - \rho_{3,flat}^*) \cdot (\rho_4^* - \rho_{4,flat}^*) \quad (19)$$

Eq.(19) is approximately valid only for small particles (with radii smaller than 1 mm). From Eq.(19) one can see that the sign of the interfacial meniscus force depends on the signs of the two last terms. If the particles are identical, or similar in a sense that for both of them $(\rho^* - \rho^*_{flat})$ has the same sign (see Fig.1.a), the force will be attractive. For dissimilar particles, for which $(\rho^* - \rho^*_{flat})$ has different signs (see Fig.1.c), the force will be repulsive.

For liquid metals the agglomeration of particles was observed on liquid metals [35-40] and at the liquid metal/slag interface [41]. From the velocity of the particles the interfacial meniscus force was estimated and the results have been found to be in good agreement with the theory.

8. THE LIQUID BRIDGE INDUCED INTERFACIAL FORCE

In Fig.1 small ceramic particles are situated on the surface of a large fluid phase 2. When the amount of the fluid phase 2 is smaller than that of the solid particles, this

fluid will form a liquid bridge between the particles (see Fig.2). Although Eq. (18) remains valid also for this case, the particular solutions will be very much different. That is why it has a sense to define this force by a different name: the “liquid bridge induced interfacial force”. This force has a major influence in liquid phase sintering [42-44].

The solution of Eq.(18) is possible only numerically, obtained for equal spheres by [45-46] and for unequal-sized spheres by [47-48]. An approximated solution exists only for equal spheres of radii r , in the limit of $V_2 \rightarrow 0$ and $x \rightarrow 0$ [45]. For the liquid bridge in gas environment [45]:

$$F_{\sigma,x} \cong -2 \cdot \pi \cdot r \cdot \sigma_{l/g} \cdot \cos \Theta \quad (20)$$

From Eq.(20) one can see that the liquid bridge at the limit $V_2 \rightarrow 0$ and $x \rightarrow 0$ will attract the particles only, if the particles are wetted by the liquid with a contact angle below 90° . The liquid bridge induced interfacial force will be weaker than that, written by Eq.(20) when the volume of the liquid bridge and the distance between the particles increase.

The liquid bridge induced interfacial force, measured experimentally in liquid metallic systems [45, 49, 50] (see also [42]) was found to be in good agreement with theory [45]. Also, the relative shrinkage of the sintered sample was found to be approximately proportional to the theoretically calculated force [46], confirming the validity of the theoretical equations [45-46].

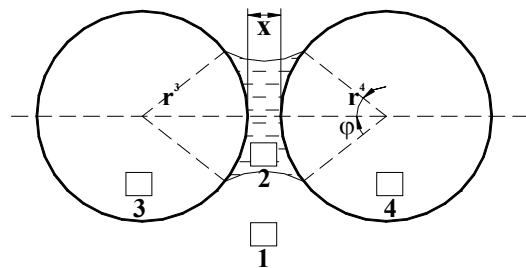


Fig.2. A liquid bridge (phase 2) between two solid particles (phases 3 and 4) in the environment of phase 1

9. SUMMARY OF INTERFACIAL FORCES

Now, let us shortly summarize the six interfacial forces, discussed in this paper. From the comparison of Eq-s (4,6.a, 10.b, 16.b-17, 19, 20) one can see that these forces are indeed different. It is obvious that these different equations will provide also different values for the interfacial forces. For one combination of materials constants the values of these six different interfacial forces are collected in Table 1.

Let us make this calculation for liquid steel at 1600°C (with $\sigma_{l/g} = 1,7 \text{ J/m}^2$ and $\rho_l = 7000 \text{ kg/m}^3$) and for solid alumina particles with $r = 10 \mu\text{m}$, $\sigma_{s/g} = 0,9 \text{ J/m}^2$, $\rho_s = 3600 \text{ kg/m}^3$, i.e. with mass of $m = 1,5 \cdot 10^{-11} \text{ kg}$. Thus, a gravity force of $F_g = mg = 1,5 \cdot 10^{-10} \text{ N}$ acts on each particle. The contact angle of steel on alumina: $\Theta = 120^\circ$. Then, from the Young equation: $\sigma_{c/l} = 1,75 \text{ J/m}^2$. Its derivatives by temperature and oxygen-concentration: $d\sigma_{c/l}/dT = -2 \cdot 10^{-4}$

J/Km^2 , $d\sigma_{c/l}/dC_O = -10 \text{ J/m}^2\text{t}\%$. The temperature and oxygen concentration gradients in liquid steel are taken as $dT/dx = 10\text{K/mm}$ and $dC_O/dx = 0,01 \text{ t}\%/\text{mm}$.

As one can see from Table 1, the majority of interfacial forces is higher than the gravity force by many magnitudes, the only exception being the interfacial meniscus force, what acts perpendicular to the gravity vector.

Table 1. Comparison of different interfacial forces*

Interfacial force type	Eq.(...)	x, μm	$F_{\sigma,x}$, N
curvature induced	4	-	$-4,4 \cdot 10^{-4}$
gradient (temperature)	5-6.a	-	$2,5 \cdot 10^{-9}$
gradient (concentration)	5-6.a	-	$1,3 \cdot 10^{-7}$
capillary	10.b	20	$-1,6 \cdot 10^{-4}$
adhesion	16.b-17	0 1	$-1,1 \cdot 10^{-4}$ $-1 \cdot 10^{-12}$
meniscus	19	100	$-1,3 \cdot 10^{-17}$
liquid bridge induced	20	0	$5,3 \cdot 10^{-5}$

* - physical data are given in the text

ACKNOWLEDGEMENT

This project has been financed by OTKA T037626.

REFERENCES

- [1] Mukai K., Lin W.: Motion of small particles in solution with an interfacial tension gradient and engulfment of the particles by solidifying interface - Tetsu to Hagané, 1994, vol. 80, pp.527-532.
- [2] Z. Wang, K. Mukai, I. J. Lee: Behaviour of fine bubbles in front of the solidifying interface - ISIJ International, 1999, vol. 39, pp. 553 – 562
- [3] G.Kaptay: Interfacial Criteria to Avoid Pushing of Particles during Solidification of Metal Matrix Composites – Materials Science Forum, 2000, vols 329-330, pp. 121-126.
- [4] G.Kaptay, K.K.Kelemen: On the force acting on a sphere moving towards a solidification front due to an interfacial energy gradient at the sphere/liquid interface – ISIJ International, 2001., vol.41., pp.305-307.
- [5] N.O.Young, J.S.Goldstein, M.J.Block: The motion of bubbles in a vertical temperature gradient - J. Fluid Mech., 1959, vol.6, pp.350-356
- [6] J. L. Anderson, M. E. Lowell, D. C. Prieve: Motion of a particle by chemical gradients Part 1. Non - electrolytes - J. Fluid Mech., 1982, vol. 117, pp. 107-121
- [7] S. C. Hardy: The motion in a vertical temperature gradient - J. of Colloid and Interface Sci., 1979, vol. 69, pp. 157–162
- [8] R. M. Merritt, R.S. Subramanian: The migration of isolated gas bubbles in a vertical temperature gradient - J. of Colloid and Interface Science, 1988, vol. 125, No 1, pp. 333 - 339
- [9] B.Braun, Ch.Ikier, H.Klein: Marangoni convection-induced motion of droplets during liquid/liquid phase separation - ESA SP-1132, 1994, vol. 4, pp. 119-122
- [10] T. A. Engh, H. Sandberg, A. Hultkvist, L. G. Norberg: Si deoxidation of steel by injection of slags with low SiO_2 activity – Scand. J. of Metallurgy, 1972, vol. 1, pp. 103-114
- [11] H. Nakae, H. Fujii, T. Shinohara, B. R. Zhao: Theoretical model of transfer of particles into molten metal - Proc. of the 9th Int. Conf. on Composite Materials, ed. By A. Miravete, 1993, pp. 255-262
- [12] G. Kaptay: Interfacial phenomena during melt processing of ceramic particle - reinforced metal matrix composites . Part

- I. Introduction of solid particles into melts - Materials Science Forum, 1996, vols. 215-216, pp. 459-466
- [13] G.Kaptay: Interfacial criteria for ceramic particle stabilised metallic foams - in: "Metal Foams and Porous Metal Structures", ed. By J.Banhart, M.F.Ashby and N.A.Fleck, MIT Verlag, 1999, pp. 141-145
- [14] S.W.Ip, Y.Wang, J.M.Toguri: Aluminum foam stabilization by solid particles - Can. Met. Quart., 1999, vol.38, pp.81-92
- [15] J.Banhart: Manufacture, characterization and application of cellular metals and metallic foams - Progress in Materials Science, 2001, vol.46, pp.559-632
- [16] B.P.Binks: Particles as surfactants - similarities and differences - Current Opinion in Colloid and Interface Science, 2002, vol.7, pp.21-42
- [17] G.Kaptay: An improved interfacial model of stabilization of metallic foams by solid particles - to be published in "Cellular Metals and Metal Foaming Technology", ed. by J.Banhart, N.A.Felck, A.Mortensen, MIT Verlag, 2003
- [18] D.Poggi, R.Minto, W.G.Davenport: Mechanism of metal entrapment in slags - J.Met., 1969, Nov., pp.40-45
- [19] H.C.Maru, D.T.Wasan, R.C.Kintner: Behavior of a rigid sphere at a liquid-liquid interface - Chem. Eng. Sci., 1971, vol.26, pp.1615-1628
- [20] K.O.Fagerlund, H.Jalkanen: Microscale simulation of settler processes in copper matte smelting - Metall. Mater. Trans., 2000, vol.31B, pp.439-451
- [21] G.Kaptay: Discussion of paper [20] - Metall. Mater. Trans. B, 2001, vol. 32B, pp.555-557
- [22] G.Kaptay: Interfacial Phenomena during Melt Processing of Ceramic Particle-Reinforced Metal Matrix Composites. Part II. Interfacial Force between a Spherical Particle and an Approaching Solid/Liquid Interface - Materials Science Forum, 1996, Vols. 215-216, pp. 467-474.
- [23] G.Kaptay: Interfacial Criteria of Spontaneous and Forced Engulfment of Ceramic Particles by an Advancing Solidification Front: Metall. Mater. Trans. A., 2001, vol.32A, pp. 993-1006.
- [24] J.H. de Boer: The influence of van der Waals forces and the primary bonds on binding energy, strength and orientation, with special reference to some artificial resins - Trans. Faraday Soc., 1936, vol.32, pp.10-38
- [25] B.V.Derjagin, N.A.Krotova, V.P.Smilga: Adhesion in Solids, Consultants Bureau, NY, 1978.
- [26] A.W.Adamson: Physical Chemistry of Surfaces, 5th ed., John Wiley and Sons Inc., NY, 1990.
- [27] R.S.Bradley: The cohesion force between solid surfaces and the surface energy of solids - Phil.Mag., 1932, vol.13, No.86, pp.853-862
- [28] H.C.Hamaker: The London-van der Waals attraction between spherical particles - Physica, 1937, vol. 10, pp. 1058-1072
- [29] G.Kaptay, K.K.Kelemen: On the Drag Force Acting on a Ceramic Particles during Production of Cast MMC-s - in: „State of the Art in Cast MMCs”, ed. by P.K.Rohatgi, TMS, 2000, pp.45-60.
- [30] G.Kaptay: Interfacial aspects to produce particulate reinforced metal matrix composites - in: „Affordable Metal-Matrix Composites for High Performance Applications”, ed. by A.B.Pandey, K.L.Kendig and T.J.Watson - TMS, Warrendale, PA, 2001, pp.71-99.
- [31] M.M.Nicolson: The interaction between floating particles - Proc. Cambridge Philos.Soc., 1949, vol.45, pp.288-295
- [32] W.A.Gifford, L.E.Scriven: On the attraction of floating particles - Chem. Eng. Sci., 1971, vol. 26, pp. 287-297
- [33] V.N. Paunov, P.A.Kralchevsky, N.D.Denkov, K.Nagayama: Lateral capillary forces between floating submillimeter particles - J. of Colloids and Interface Science, 1993, vol. 157, pp. 100-112
- [34] D.Y.Chan, J.D.Henry, JR., L.R.White: The interaction of colloidal particles collected at fluid interfaces - J. of Colloid and Interface Sci., 1981, vol. 79, pp. 410-418
- [35] V.A.Deryabin, S.I.Popel, N.L.Baranova: Kapillyarnoe vzaimodejstvie chastic na poverhnosti zhidkosti - Adgeziya rasplavov i pajka materialov, 1979, pp. 122-128
- [36] H.Yin, H.Shibata, T.Emi, M.Suzuki: "In-situ" observation of collision, agglomeration and cluster formation of alumina inclusion particles on steel melts - ISIJ International, 1997, vol. 37, pp. 936-945
- [37] H.Yin, H.Shibata, T.Emi, M.Suzuki: Characteristics of agglomeration of various inclusion particles on molten steel surface - ISIJ International, 1997, vol. 37, pp. 946-955
- [38] H.Shibata, H.Yin, T.Emi: The capillary effect promoting collision and agglomeration of inclusion particles at the inert gas-steel interface - Phil. Trans. R. Soc, Lond., 1998, vol. 356, pp. 957-966
- [39] K.Nakajima, S.Mizoguchi: Capillary interaction between inclusion particles on the 16 Cr stainless steel melt surface - Metall. Mater. Trans., 2001, vol. 32B, pp. 629-641
- [40] S.Kimura, K.Nakajima, S.Mizoguchi: Behaviour of alumina-magnesia complex inclusions and magnesia inclusions on the surface of molten low-carbon steels - Metall. Mater. Trans., 2001, vol. 32B, pp. 79-85
- [41] P. Misra, V.Chevrier, S.Sridhar, A.W.Cramb: In-situ observations of inclusions at the (Mn, Si)-killed steel/CaO-Al₂O₃ interface - Metall. Mater. Trans., 2000, vol. 31B, pp. 1135-1139
- [42] V.N.Eremenko, Y.V.Naidich, I.A.Lavrinenko: Liquid Phase Sintering, Consultants Bureau, New York, 1970.
- [43] R.M.German: Liquid Phase Sintering, Plenum Press, New York, 1985
- [44] A.Mortensen: Kinetics of densification by solution-reprecipitation - Acta Mater., 1997, vol. 45, pp. 749-758
- [45] Iu.V.Naidich, I.A.Lavrinenko: Capillary adhesion forces between solid particles with an intermediate liquid layer at the contact. Poroshkovaya Metallurgiya, 1965, vol.34, No.10, pp.61-66
- [46] W.J.Huppman, H.Riegger: Modelling of rearrangement process in liquid phase sintering - Acta Metall., 1975, vol.23, pp.965-971
- [47] V.P.Mehrotra, K.V.S.Sastry: Pendular bond strength between unequal-sized spherical particles - Powder Technology, 1980, vol.25, pp.203-214
- [48] V.P.Mehrotra, K.V.S.Sastry: A novel method for the calculation of pendular bond characteristics between unequal-sized particles - Powder Technology, 1985, vol.41, pp.259-263
- [49] S.I.Popel, V.A.Deriabin, Iu.A.Deriabin: Sili, stiagivaiushie chastici manzhetami rasplavov - in: "Metodi issledovaniia I svoistva granic razdela kontaktiruiushich faz", Kiev, Naukova Dumka, 1977, pp.3-18
- [50] V.A.Deriabin, S.I.Popel, N.V.Ratnikova: Kapilliarnoie sceplenie chastic cherez metallicheskie prosloiki - Adgeziya rasplavov i paika materialov, 1983, No.11, pp.12-16

Author: George Kaptay: Professor, Head of Department, University of Miskolc, Dep. of Physical Chemistry, 3515 Hungary, Miskolc, Egyetemvaros, e-mail: kaptay@hotmail.com